B. Sc. Part II (Hons) 4th Paper

14/01/22

Exact diff. equns (contd.)

Method 3 If $Mx - Ny \neq 0$ and the equation $M\,dx + N\,dy = 0$ can be written as

$$y f_1(xy)\,dx + x f_2(xy)\,dy = 0$$ then

$$\frac{1}{Mx - Ny} = IF.$$

Q. Solve $y(xy + 2x^2y^2)\,dx + x(xy - x^2y^2)\,dy = 0$

Soln

$$M = y(xy + 2x^2y^2)$$
$$N = x(xy - x^2y^2).$$

$$\therefore Mx - Ny = xy(xy + 2x^2y^2) - xy(xy - x^2y^2) = 3x^3y^3 \neq 0$$

$$\therefore IF = \frac{1}{3x^3y^3}.$$

Multiplying the given eqn by IF, we have

$$\frac{xy^2(1 + 2xy)\,dx}{3x^3y^3} + \frac{x^2y(1 - xy)}{3x^3y^3}\,dy = 0$$

$$\Rightarrow \frac{1 + 2xy}{3x^2y}\,dx + \frac{1 - xy}{3xy^2}\,dy = 0 \quad \left[\text{Multiply both sides by } 3\right]$$

$$\Rightarrow \frac{dx}{x^2y} + \frac{2\,dx}{x} + \frac{dy}{xy^2} - \frac{dy}{y} = 0$$

$\Rightarrow \frac{1}{y} d\left(\frac{-1}{x}\right) + 2\frac{dx}{x} + \left(-\frac{1}{x}\right) \cdot d\left(\frac{1}{y}\right) - \frac{dy}{y} = 0$

$\Rightarrow d\left(-\frac{1}{x}\cdot\frac{1}{y}\right) + 2\frac{dx}{x} - \frac{dy}{y} = 0$

Integrating we get

$\Rightarrow -\frac{1}{xy} + 2\log x - \log y = \log k$

$\Rightarrow \log x^2 = \log ky + \frac{1}{xy}$

$\Rightarrow x^2 = ky \cdot e^{\frac{1}{xy}}$

Q. Solve $(1-yx)y\,dx + (1+yx)x\,dy = 0$.

Soln $M = y(1-xy)$, $N = x(1+xy)$

$\therefore Mx - Ny = xy(1-xy) - xy(1+xy) = -2x^2y^2 \neq 0$

$\therefore IF = \frac{1}{Mx-Ny} = -\frac{1}{2x^2y^2}$.

Multiplying the given eqn with IF, we've

$\frac{1-xy}{x^2y}dx + \frac{1+xy}{xy^2}dy = 0$

$\Rightarrow \frac{1}{x^2y}dx - \frac{dx}{x} + \frac{1}{xy^2}dy + \frac{dy}{y} = 0$

$\Rightarrow d\left(-\frac{1}{xy}\right) - \frac{dx}{x} + \frac{dy}{y} = 0$ Integrating, we get

$\Rightarrow -\frac{1}{xy} - \log x + \log y = \log k$